

Self-sustained current oscillations in spin-blockaded quantum dots

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Self-sustained current oscillation observed in spin-blockaded double quantum dots is explained as a consequence of periodic motion of dynamically polarized nuclear spins (along a limit cycle) in the spin-blockaded regime under an external magnetic field and a spin-transfer torque. It is shown, based on the Landau-Lifshitz-Gilbert equation, that a sequence of semistable limit cycle, Hopf and homoclinic bifurcations occurs as the external field is tuned. The divergent period near the homoclinic bifurcation explains well why the period in the experiment is so long and varies by many orders of magnitudes.

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Quantum dots, also known as artificial atoms, have many properties of natural atoms such as discrete energy levels and shell structures [1, 2]. The physics involved in quantum dots is very rich because of tunability and comparability of three energy scales: level spacing, Coulomb interaction, and thermal energy. Unlike natural atoms, quantum dots allow transport measurements. Many interesting transport phenomena like resonant tunnelling, Coulomb blockade, spin-blockade, Kondo effects, quantum conductance etc. have been observed and explained. Applications in nano-electronics, spintronics and quantum computing due to possible long coherence time have been proposed and in some cases implemented. It is known that both the nuclear and electron spin degrees of freedom in semiconductor nanostructures can be manipulated by the hyperfine interactions [3–6]. In the endeavor of using hyperfine interaction to manipulate the electron transport in quantum dots, one long-term unexplained intriguing phenomenon was observed by Ono and Tarucha in 2004 [6] in spin-blockaded double quantum dots: tunneling current I under a DC-bias, as artistically shown in Fig. 1, oscillates with a period up to minutes under certain conditions. In this letter, we show that dynamically polarized nuclear spins (DPNS), governed by the generalized Landau-Lifshitz-Gilbert equation, can oscillate with time under a DC-bias in a magnetic field window. The back effect of the DPNS oscillation on the electron tunneling leads to the experimentally observed self-sustained current oscillation.

A coherent theory should be consistent with the following experimental findings: 1) The oscillations observed only in the spin-blockaded regime in a magnetic field window accompany by a current jump. 2) Both the period and amplitude increase with the field before the oscillation disappears. 3) The oscillation period can be tuned by the magnetic field by several orders of magnitude from seconds (the instrumental limit in the experiment) up to several minutes. 4) The oscillation is closely related to the motion of dynamically polarized nuclear spins (DPNS) [6–10]. It is clear that the oscil-

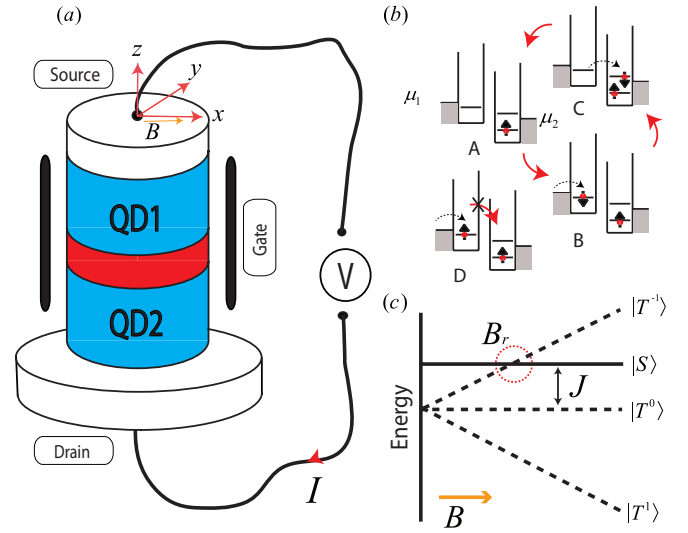


FIG. 1: (Color online) (a) Schematic setup of spin-blockaded double quantum dots device. (b) Four possible configurations in spin-blockaded regime. One spin-up electron is in the right quantum dot below the chemical potentials in configuration A. In configuration B, one electron on each of two dots form a spin singlet state. Configuration C is the spin singlet state of two electrons in the right dot. Spin-blockaded configuration D is a spin triplet state of two dots with one electron on each. (c) Schematic of field-dependence of triplet and singlet states. Triplet state $|T^{-1}\rangle$ cross with singlet state $|S\rangle$ near the resonance field B_r (red circle).

lation theory for semiconductor superlattices [11] is not applicable because the period would be order of $100ns$ electron tunneling time, instead of observed seconds and minutes. Thermal and impurity effects were also ruled out [6]. Although the nuclear spins in a quantum dot is known to have a very long (up to seconds) relaxation time, the relaxation process is not a periodic motion so that it cannot be the cause of the oscillation.

In order to construct a sensible model, let us examine the plausible microscopic process of electrons and nuclei in spin-blockaded double quantum dots. In the experiment [6], two vertical disk-like quantum dots (InGaAs-

AlGaAs multilayer structure) are weakly connected in series between source and drain (illustrated in Fig. 1a). Four possible configurations in the spin-blockaded regime are shown in Fig. 1b: One electron is trapped in the right dot (configuration A). The second electron, hopping from the source lead to the left dot, forms either spin singlet (B) or triplet (D) states. Electron tunneling cycle $A \rightarrow B \rightarrow C \rightarrow A$ is allowed while tunneling is blocked in D, here C is spin singlet state of two electrons on the right dot. As shown in Fig. 1c, three triplet states ($|T^0\rangle$, $|T^{\pm 1}\rangle$) are degenerated in the absence of an external magnetic field, and are below the spin singlet state $|S\rangle$ by an energy J [12], order of inter-dot exchange energy. Due to the spin blockade and weak coupling of dots with two leads, a leakage current of order of 1pA, corresponding to 100ns electron tunneling time, exists. An external field lifts the triplet degeneracy, and $|S\rangle$ and $|T^{-1}\rangle$ states shall anti-cross around certain field. The current experiences a jump near the crossing field B_r because spin blockade is partially removed by spin flipping there, and transition from configuration D to B becomes possible. It is known that this spin-flip process dynamically polarizes the nuclear spins, resulting in DPNS with a magnetization \vec{M} [13]. This spin-flip process mediates also an effective spin transfer from electron spins to the nuclear spins. Adopting the view that dynamical motion of DPNS is responsible to the observed self-sustained current oscillation, we concentrate on the DPNS dynamics under the influence of both magnetic field and the above spin-flip process mediated spin transfer torque.

It is well-known that the Landau-Lifshitz-Gilbert equation governs the generic dynamics of a macro-spin preserving its magnitude while the so-called Landau-Lifshitz-Bloch equation is for the dynamics of a macro-spin whose magnitude can also vary. For simplicity and our belief that magnitude change of DPNS magnetization is irrelevant to the observed oscillation, we assume following dynamics for $\vec{m} = \vec{M}/|\vec{M}|$ [14],

$$\frac{d\vec{m}}{dt} = -\vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{d\vec{m}}{dt} + a \vec{m} \times (\vec{m} \times \hat{x}). \quad (1)$$

Here t is in the units of $(\gamma M)^{-1}$, order of submicron second for GaAs with $M = 10^6 A/m$ and the nuclear gyromagnetic ratio $\gamma = 10(A \cdot s/m)^{-1}$ [15]. The first term on the right-hand side of Eq. (1) describes the Larmor precession around the effective field $\vec{H}_{eff} = \vec{H} - D\vec{m}$ from both external magnetic field \vec{H} along x-axis, and demagnetization field $D\vec{m}$ (in the units of $|M|$). Cylindrical disk-like dots in experiment [6] have diagonal demagnetization factors with $D_x = D_y = 0$ and $D_z = 1$. In reality, a small difference between D_x and D_y exist either due to the inhomogeneity or deviation of dots from the perfect cylindrical shape, our numerical results show that the physics reported here remain the same for $D_x \neq D_y$. The second term is the phenomenological Gilbert damping with a dimensionless constant α . The third term

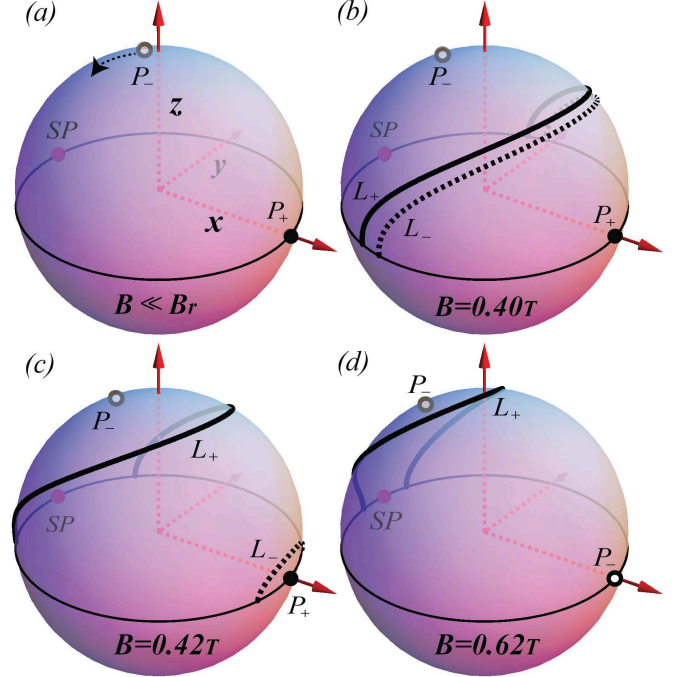


FIG. 2: (Color online) Change of attractors as an external magnetic field varies: Only upper sphere was showed for clarity. Solid (dash) black line L_+ (L_-) denotes stable (unstable) limit-cycles (LCs). P_+ (P_-) labels stable (unstable) fixed points (FPs), and red dot SP is the saddle point at $\vec{m} = (-1, 0, 0)$. (a) For $B \ll B_r$ and the only stable attractor is P_+ ; (b) $B = 0.40T$, a slightly over the semistable LC bifurcation where L_+ and L_- are generated; (c) $B = 0.42T$, slightly below subcritical Hopf bifurcation field at which L_- merges with P_+ and becomes an unstable FP; (d) $B = 0.62T$, slightly below the homoclinic bifurcation field at which L_+ touches SP.

is the Slonczewski torque (per spin) along x-direction [16] originated from the transition from $|T^{-1}\rangle$ to $|S\rangle$ mentioned earlier (Fig. 1c). The Slonczewski coefficient $a = \eta W/N$ is proportional to transition rate W from $|T^{-1}\rangle$ to $|S\rangle$ and inversely proportional to the polarized nuclear number $N \sim 10^5 - 10^6$ [6]. Dimensionless coefficient $\eta = S_{\perp}/\hbar$ measures the average spin angular momentum quanta transferred from one electron to nuclei. W in the Fermi golden rule approximation reads [8, 17, 18]

$$W = W_0 \frac{\Gamma^2}{\Delta E^2 + \Gamma^2} \times \xi \quad (2)$$

Where

$$\xi = \begin{cases} 1, & \Delta E > 0 \\ \exp(\frac{\Delta E}{k_B T}), & \Delta E < 0 \end{cases}$$

$\Delta E \equiv |g\mu_B B| - J$ is the level spacing between $|T^{-1}\rangle$ and $|S\rangle$ states. The effective Lande g-factor for GaAs is $g = -0.44$ [19]. k_B is the Boltzmann constant and $T = 1.8K$ is the experimental temperature. The level broadening Γ of state $|T^{-1}\rangle$ is order of phonon energy of

μeV [18], and $W_0 \sim 10^3 - 10^4$ [20] is the typical resonant spin flip-flop rate.

In the absence of the external field, all points on the equator of \vec{m} -sphere are marginal stable fixed points (FPs) while $\vec{m} = (0, 0, \pm 1)$ are unstable. Under a very small field, $\vec{m} = (-1, 0, 0)$ becomes the only saddle point (SP in Fig. 2) and $\vec{m} = (1, 0, 0)$ the only stable attractor (P_+ in Fig. 2). The two unstable FPs (upper P_- in Fig. 2) move toward SP along the big cycle in $x - z$ plane as B increases (indicated by the arrow in Fig. 2a). In terms of energy, α -term is always an energy sinker while a -term could be both energy sinker and source [14]. In the vicinity of P_+ , α -term serves as an energy source that can be seen from the fact that a -term tends to push the system away from P_+ . As B approaching B_r , a -term may become large enough to destabilize P_+ . This is confirmed by the standard stability analysis [21] by calculating the Liapunov exponent at P_+ . The Liapunov exponents at P_+ become positive at about $B = 0.42T$, and the system has no stable FPs at this point. This entails the existence of limit-cycle(s) (LCs) in two dimensions (current case). Indeed, our numerical calculations support this scenario. Fig. 2 shows the locations of various attractors of Eq. 1 at various B . For a field much smaller than B_r (Fig. 2a), all phase flows end at P_+ , the only stable FP of the system. At a critical field B_0 , slightly smaller than $0.40T$, the system undergoes a semistable LC bifurcation (shown in Fig. 2b at which a pair of LCs, one stable (solid black line) and the other unstable (dashed line), appear simultaneously. The two LCs move in opposite direction as the field increases further. At a critical value $B = B_1$ around $0.42T$, the unstable LC merges with P_+ , undergoing a subcritical Hopf bifurcation and turning P_+ into an unstable FP P_- . DPNS changes from a static state to a LC state, the onset of self-sustained oscillation (Fig. 2c). The LC touches the saddle point at another critical value $B = B_2$ of about $0.62T$, undergoing a homoclinic bifurcation (Fig. 2d). The corresponding period diverges when B goes to B_2 because the phase velocity vanishes at the saddle point. These are exactly what were observed in the experiment [6].

In order to obtain the field-dependence of period, one needs to locate first the LC for a given field. This can be done by using the Melnikov theory [21, 22] valid for a nearly conserved system of small α and a . It is well known that the energy dissipation rate [23]

$$f(U) \equiv \oint_{L(U)} \left(-\alpha \frac{d\vec{m}}{dt} - a\vec{m} \times \hat{x} \right) \cdot d\vec{m} \quad (3)$$

is a function of equal energy contour $L(U)$, thus also a function of energy $U \equiv \frac{1}{2}(D_x m_x^2 + D_y m_y^2 + D_z m_z^2) - \vec{H} \cdot \vec{m}$. f is the dissipated energy after the system moves along $L(U)$ once. Interestingly, the Melnikov function of stable and unstable FPs and LCs is zeros by definition. According to the Melnikov theory, the LC is approximated by

one particular $L(U^*)$ that satisfies $f(U^*) = 0$. Fig. 3a is the U -dependence of $f(U)$ for $B = 0.40T, 0.42T, 0.62T$ (from top to bottom) in the vicinity of B_0, B_1 and B_2 respectively. By definition, the energy contours for energy extremes (P_{\pm} minima and maxima) are points, thus the Melnikov function is zero there. These correspond to the far left and far right nodes in the figure. The origin (crossing of x - and y -axis) was chosen to be the energy of the saddle point. f -curve have positive (negative) slopes are stable (unstable). To see this, consider a node U^* with $\frac{\partial f}{\partial U}|_{U^*} < 0$ and a slightly deviation of U from U^* , say $U > U^*$, U should decrease after the system moving along the $L(U)$ once because $f(U) < f(U^*) = 0$. Thus the system tends to push U back to U^* . For $B = 0.40T$, zero slop of $f(U)$ at the node around $U = 0.1$ corresponds to generation of a pair of LCs, leading to a semistable LC bifurcation ($f(U^*) = 0, \frac{\partial f}{\partial U}|_{U^*} = 0$). A slight increase of B lowers $f(U)$ curve near $U^* = 0.1$ and the node splits into two. The left (right) one with negative (positive) slope corresponds to stable (unstable) LC. The stable LC L_+ moves towards the SP while unstable LC L_- moves towards P_+ as B increases. The L_- merges with P_+ around $0.42T$, and turns P_+ into an unstable FP P_- (negative slop). This is a subcritical Hopf bifurcation. Further increase of B to $0.62T$, L_+ touches saddle point SP and become a homoclinic loop, resulting in a homoclinic bifurcation. After locating the LC for a given field in the window of $[B_1, B_2]$, the period of DPNS can be evaluated by

$$\tau = \oint_{L(U^*)} \frac{d\vec{m}}{\frac{d\vec{m}}{dt}}, \quad (4)$$

where $d\vec{m}/dt$ is given by Eq.(1). As shown in Fig. 3b, the period increases monotonically with field and diverges at B_2 .

The electron in triplet state ($|T^{-1} \rangle$) plays dual roles. It not only glues together the nuclear spins through the hyperfine coupling so that nuclear spins become polarized, but also generates a torque on the DPNS. The self-sustained current oscillation comes from the periodic motion of DPNS along an LC. The LC originates from the instability of a static DPNS state under two competing forces: One is energy input from the tunneling electrons that drives the DPNS away from its static state, a FP. The other is the dissipation of α -term that tends to push the DPNS to its FP. The periodical motion of DPNS leads to the current oscillation. This theory explains why the current oscillation was only observed in the spin-blockaded regime and under an magnetic field. It also explains why the period can be several order of magnitude longer than the typical spin precession period. In principle, the current oscillation reported here should exist in systems with more than two dots in the spin-blockaded regime. However, it may be easier to observe it in a spin-blockaded double quantum dots because it is

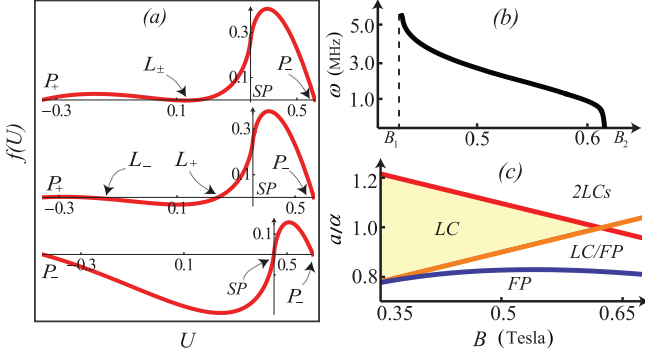


FIG. 3: (Color online) (a) The Melnikov curves $f(U)$ for $B = 0.40T$, $0.42T$, and $0.62T$ (from top to bottom). Zeros with positive (negative) slope are stable (unstable) attractors. The far left node corresponds to $(1, 0, 0)$, while the far right node $(0, 0, \pm 1)$ and the origin (crossing of x - and y -axis) was chosen to be the energy of SP. (b) Field dependence of SSCO frequency ω . The period diverges at B_2 . (c) Phase diagram in $B - a/\alpha$ plane. Red, orange, and blue curves are for homoclinic, subcritical Hopf, and semistable LC bifurcations, respectively. FP, LC, LC/FP, and 2LCs denote fixed-point, one limit-cycle, co-existence of limit-cycle and fixed-point, and two limit-cycles phases, respectively.

easier to glue fewer nuclear spins in a smaller space. This is in a sharp contrast to the current oscillation observed in superlattices that originates from the negative differential resistance [14], where it is only observed in superlattices with more than 20 wells. It shall be interesting to see whether there are also field-induced Hopf and homoclinic bifurcations in Landau-Lifshitz-Bloch dynamics that governs spin dynamics whose magnitude can also vary.

Interestingly, if a in Eq. 1 can vary independently from the field B , one can obtain the homoclinic bifurcation, subcritical Hopf bifurcation, and semistable LC bifurcation curves as a function of B and a/α in a similar way as what we explained earlier. Our results are showed in Fig. 3c. The phase diagram shows that Eq. 1 supports various stable phases (stable attractors), including stable FPs only, coexistence of a stable FP and a stable LC, and existence of two stable LCs. In summary, there are a number of predictions in our theory to be confirmed. According to our analysis, a LC solution can only exist when spin transfer torque is large enough and self-sustained current oscillation appears and disappears when the tunneling current varies. The current can be controlled by the electron coupling between electrodes and dots through gates. Thus, this provides an experimental way of testing our theory. Also, our theory predicts multiple stable attractors (Fig. 3c) in certain parameter regions, either the coexistence of LC and FP or the existence of two LCs. Thus one should expect hysteresis loops or two oscillation periods if proper conditions are satisfied.

In conclusion, we showed that an external magnetic

field can induce a Hopf bifurcation at a low field and a homoclinic bifurcation at a high field for DPNS in a spin-blockaded double quantum dots, between which is the magnetic field window for the self-sustained current oscillation. The amplitude and period of the oscillation becomes bigger and bigger as the field increases, in good agreement with the experimental findings. The period diverges at the homoclinic bifurcation in our model which explains well why the period is of several orders of magnitude larger than the fundamental time scales.

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- [1] M.A. Kastner, Phys. Today 46, 24 (1993).
- [2] T. Brandes, Phys. Rep. 408, 315 (2005).
- [3] Semiconductor Spintronics and Quantum Computation, edited by D. D. Awschalom, N. Samarth, and D. Loss (Springer-Verlag, Berlin, 2002).
- [4] T. Fujisawa, D.G. Austing, Y. Tokura, Y. Hirayama and S. Tarucha, Nature (London) **419**, 278 (2002); R. Hanson, B. Witkamp, L.M.K. Vandersypen, L.H. Willems van Beveren, J.M. Elzerman, and L.P. Kouwenhoven, Phys. Rev. Lett. **91**, 196802 (2003).
- [5] M. Dobers, K. Vonklitzing, and G. Weimann, Phys. Rev. B **38**, 5453 (1988).
- [6] K. Ono and S. Tarucha, Phys. Rev. Lett. **92**, 256803 (2004).
- [7] T. Inoshita and S. Tarucha, Physica E **22**, 422 (2004).
- [8] C. Lopez-Monis, J. Inarrea, and G. Platero, New. J. Phys. **13**, 053010 (2011).
- [9] M.S. Rudner, and L.S. Levitov, Phys. Rev. Lett. **99**, 036602 (2007).
- [10] M.S. Rudner, F.H.L. Koppens, J.A. Folk, L.M.K. Vandersypen, L.S. Levitov, Phys. Rev. B **84**, 075339 (2011).
- [11] X.R. Wang, and Q. Niu, Phys. Rev. B **59**, R12755 (1999); X.R. Wang, J.N. Wang, B.Q. Sun, and D.S. Jiang, Phys. Rev. B **61**, 7261 (2000); Z.Z. Sun, H.T. He, J.N. Wang, S.D. Wang, and X.R. Wang, Phys. Rev. B **69**, 045315 (2004).
- [12] S. Yin, Q.F. Sun, Z.Z. Sun, and X.R. Wang, J. Phys.: Condens. Matter **17**, L183L189 (2005).
- [13] M. Gullans et al., Phys. Rev. Lett. **104**, 226807 (2010).
- [14] Z.Z. Sun and X.R. Wang, Phys. Rev. Lett. **97**, 077205 (2006); X.R. Wang and Z.Z. Sun, *ibid* **98**, 077201 (2007).
- [15] E. V. Charnaya et al., Phys. Rev. Lett. **88**, 097602 (2002).
- [16] J. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
- [17] T. Fujisawa et al., Science **282**, 932 (1998).
- [18] J. Inarrea, G. Platero, and A. H. MacDonald, Phys. Rev. B **76**, 085329 (2007).
- [19] J. R. Petta et al., Science **309**, 2180 (2005).
- [20] J. Baugh, Y. Kitamura, K. Ono, S. Tarucha, Phys. Rev. Lett. **99**, 096804 (2007).
- [21] S.H. Strogatz, *Nonlinear Dynamics and Chaos*, (Addison-Wesley, 1994); L. Perko, *Differential Equations and Dynamical Systems*, (Springer, 2001).
- [22] G. Bertotti et al., Phys. Rev. Lett. **94**, 127206 (2005).
- [23] Z.Z. Sun and X.R. Wang, Phys. Rev. B **73**, 092416 (2006).

- (2006); *ibid* **74**, 132401 (2006).
- [24] L. Landau, and E. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1953); T.L. Gilbert, Phys. Rev. **100**, 1243 (1955).
- [25] I.A. Merkulov, A.L. Efros, and M. Rosen, Phys. Rev. B **65**, 205309 (2002).